① $f(x) = 3x^5 - 5x^3 + 2$

$f'(x) = 15x^4 - 15x^2$

$f'(x) = 0 = 15x^2(x^2-1)$

$0 = 15x^2(x-1)(x+1)$

$x = 0, 1$ and -1

$f'(x)$	+		−		−		+
$f(x)$	inc	-1	dec	0	dec	1	inc

a) $\boxed{f \text{ is increasing on } (-\infty,-1) \cup (1,\infty)}$

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$f'(x) = 15x^4 - 15x^2$

$f''(x) = 60x^3 - 30x$

$f''(x) = 30x(2x^2-1)$

$f''(x) = 0 = 30x(2x^2-1)$

$30x = 0 \qquad 2x^2 - 1 = 0$

$x = 0 \qquad x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$

$f''(x)$	−		+		−		+
$f(x)$	c.d	$-1/\sqrt{2}$	c.u.	0	c.d	$1/\sqrt{2}$	c.u

$\boxed{f \text{ is concave up on } \left(\frac{-1}{\sqrt{2}}, 0\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)}$

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c) tangent line when $x = 1$

$f(1) = 3(1)^5 - 5(1)^3 + 2 = 0 \leftarrow$ y coord of point

$f'(1) = 15(1)^4 - 15(1)^2 = 0 \leftarrow$ slope

$\boxed{y = 0 \text{ is the equation of the tangent line when } x = 1}$

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② $f(x) = xe^{1-x}$

$f'(x) = xe^{1-x}(-1) + e^{1-x} \cdot 1$

$f'(x) = e^{1-x}(-x+1)$

$f'(x) = 0 = e^{1-x}(-x+1)$

$e^{1-x} \neq 0$ no critical point

$0 = -x + 1$

$x = 1$

$f'(x)$	+		−
$f(x)$	inc	1	dec

a) $\boxed{f \text{ is increasing on } (-\infty, 1)}$

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b) Range = set of all possible y values.

Part (a)

Y value of local max $\quad f(1) = e^{1-1}(1) = 1$

Range $(-\infty, 1]$

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c) $f'(x) = e^{1-x}(-x+1)$

$f''(x) = e^{1-x}(-1) + (-x+1)e^{1-x}(-1)$

$= e^{1-x}(-1 + x - 1)$

$f''(x) = e^{1-x}(x-2)$

$f''(x) = 0 = e^{1-x}(x-2)$

$e^{1-x} \neq 0$

$x - 2 = 0$

$x = 2$

$f''(x)$	−		+
$f(x)$	cd	2	cu

P.O.I $(2, 2e^{-1})$

Oct 5-10:42 AM

③ $f(x) = 3x^4 + x^3 - 21x^2$

a) $f'(x) = 12x^3 + 3x^2 - 42x$

$f'(2) = 24 \leftarrow$ slope at $(2,-28)$

$\boxed{\text{Equation of tangent line: } y + 28 = 24(x-2)}$

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b) Abs. min will occur either at a local min. or at endpoint. Since the domain is $(-\infty, \infty)$, there is no endpoint so we need a local min.

$f'(x) = 12x^3 + 3x^2 - 42x$

$= 3x(4x^2 + x - 14)$

$f'(x) = 3x(4x-7)(x+2)$

$f'(x) = 0 = 3x(4x-7)(x+2)$

$x = 0, -2$ and $7/4$

$f'(x)$: $-$ | -2 | $+$ | 0 | $-$ | $7/4$ | $+$

$f(x)$: dec | -2 | inc | 0 | dec | $7/4$ | inc

Local mins at $x = \frac{7}{4}$ and $x = -2$

$f(7/4) = -30.8$

$f(-2) = -44 \leftarrow$ smallest

$\boxed{\text{Abs. minimum value of } f = -44}$

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b) $f'(x) = 12x^3 + 3x^2 - 42x$

$f''(x) = 36x^2 + 6x - 42$

$= 6(6x^2 + x - 7)$

$f''(x) = 6(6x+7)(x-1)$

$f''(x) = 0 = 6(6x+7)(x-1)$

$x = -7/6$ or 1.

$f''(x)$: $+$ | $-7/6$ | $-$ | 1 | $+$

$f(x)$: c.u. | $-7/6$ | c.d. | 1 | c.u.

$\boxed{\text{function } f \text{ has points of inflection when } x = -\frac{7}{6} \text{ and } x = 1}$

Oct 5-11:20 AM

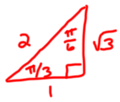
④ $f(x) = 2\cos x + x$ $\quad [0, 2\pi]$

a) $f'(x) = -2\sin x + 1$

$f'(x) = 0 = -2\sin x + 1$

$\frac{1}{2} = \sin x$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$



$f'(x)$: $+$ | $\frac{\pi}{6}$ | $-$ | $\frac{5\pi}{6}$ | $+$

$f(x)$: 0 inc | $\frac{\pi}{6}$ | dec | $\frac{5\pi}{6}$ | inc 2π

$\boxed{f \text{ is decreasing on } \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)}$

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b) $f'(x) = -2\sin x + 1$

$f'(\frac{\pi}{2}) = -2\sin\frac{\pi}{2} + 1$

$f'(\frac{\pi}{2}) = 1 \leftarrow$ slope

$f(x) = 2\cos x + x$

$f(\frac{\pi}{2}) = 2\cos\frac{\pi}{2} + \frac{\pi}{2}$

$f(\frac{\pi}{2}) = \frac{\pi}{2} \leftarrow y$ coordinate

$\boxed{\text{Equation of tangent line: } y - \frac{\pi}{2} = x - \frac{\pi}{2} \quad \text{OR} \quad y = x}$

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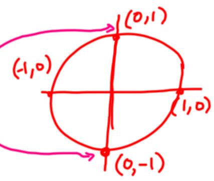
c) $f'(x) = -2\sin x + 1$

$f''(x) = -2\cos x$

$f''(x) = 0 = -2\cos x$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$



$f''(x)$: $-$ | $\frac{\pi}{2}$ | $+$ | $\frac{3\pi}{2}$ | $-$

$f(x)$: 0 c.d | $\frac{\pi}{2}$ | c.u. | $\frac{3\pi}{2}$ | c.d 2π

$\boxed{\text{Points of inflection when } x = \frac{\pi}{2} \text{ and } x = \frac{3\pi}{2}}$

Oct 5-11:29 AM

⑤ $f(x) = \ln(2+\sin x) \qquad 0 \le x \le 2\pi$

a) $f'(x) = \frac{1}{2+\sin x} \cdot \cos x$

$f'(x) = \frac{\cos x}{2+\sin x}$

Critical points when $f'(x)=0$ or $f'(x)$ undefined.

$f'(x) = 0$ when $\cos x = 0$
$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$f'(x)$ undefined if $2+\sin x = 0$
$\sin x = -2$
which never happens.

$f'(x)$: + | − | +
$f(x)$: 0 inc $\pi/2$ dec $3\pi/2$ inc 2π

$f(\pi/2) = \ln(\sin\frac{\pi}{2}+2) = \ln 3 \leftarrow$ local max value of f.

$f(\frac{3\pi}{2}) = \ln(\sin\frac{3\pi}{2}+2) = \ln 1 = 0 \leftarrow$ local min. value of f

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b) Abs. max/min occur either at local max/mins found in part (a) OR at endpoints of the domain.

$f(0) = \ln(2+\sin 0) = \ln 2$
$f(2\pi) = \ln(2+\sin 2\pi) = \ln 2$
not greater than $\ln 3$, or less than zero.

$\boxed{\begin{array}{l}\text{Abs. Min Value} = 0 \\ \text{Abs. max value} = \ln 3\end{array}}$

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$f'(x) = \frac{\cos x}{2+\sin x}$

$f''(x) = \frac{(2+\sin x)(-\sin x) - \cos x(\cos x)}{(2+\sin x)^2}$

$= \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$

$= \frac{-2\sin x - (\sin^2 x + \cos^2 x)}{(2+\sin x)^2}$

$f''(x) = \frac{-2\sin x - 1}{(2+\sin x)^2}$

NEXT SLIDE →

Oct 5-3:14 PM

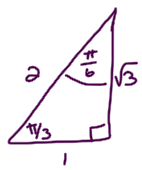
$f''(x) = \frac{-2\sin x - 1}{(2+\sin x)^2}$

$f''(x) = 0$

$0 = -2\sin x - 1$

$1 = -2\sin x$

$-\frac{1}{2} = \sin x$



ref ∡ $= \frac{\pi}{6}$ but quadrants III & IV

$x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$

$f''(x)$: − | + | −
$f(x)$: 0 c.d $7\pi/6$ c.u $\frac{11\pi}{6}$ c.d. 2π

Points of inflection when $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$

Oct 5-3:17 PM